

# *Filtrage fréquentiel*

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$I(x,y)$  image originale

$$F(u,v) = TF[I(x,y)]$$

$R(x,y)$  image filtrée

$$G(u,v) = TF[R(x,y)]$$

$$G(u,v) = H(u,v) \cdot F(u,v)$$

multiplication  
élément par élément

$H(u,v)$ : *filtre*  
*fonction de transfert du filtre*

*exemples:*

*Filtre passe-haut idéal*

$$H(u,v) = \begin{cases} 0 & \text{si } D(u,v) < D_0 \\ 1 & \text{si } D(u,v) \geq D_0 \end{cases}$$

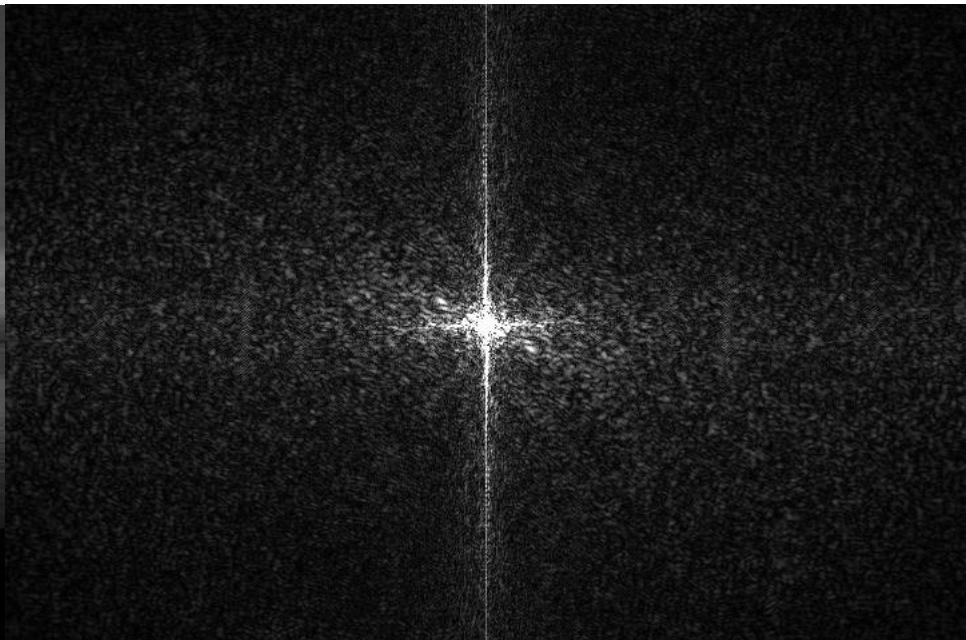
*Filtre passe-bas idéal*

$$H(u,v) = \begin{cases} 1 & \text{si } D(u,v) < D_0 \\ 0 & \text{si } D(u,v) \geq D_0 \end{cases}$$

*Image originale*



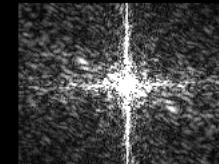
*Spectre d'amplitude*



*Image filtrée*



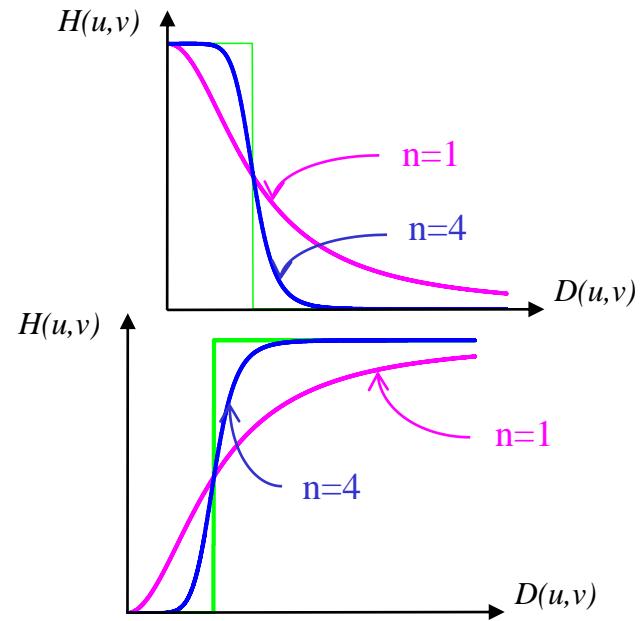
*filtre «passe-bas »*



## Filtres de Butterworth:

**passé-bas:** 
$$H(u,v) = \frac{1}{1 + (D(u,v)/D_0)^{2n}}$$

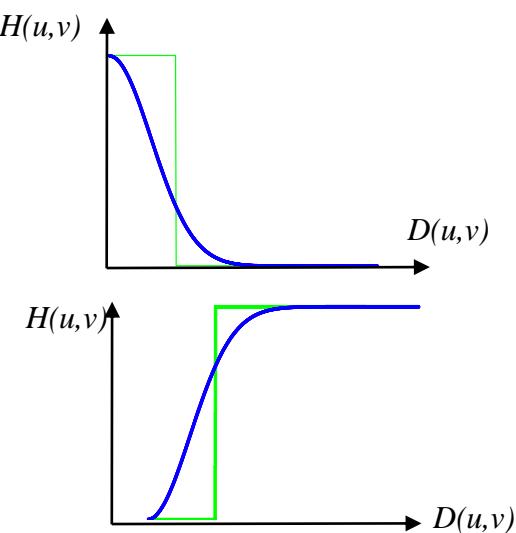
**passé-haut:** 
$$H(u,v) = \frac{1}{1 + (D_0/D(u,v))^{2n}}$$



## Filtres gaussiens:

**passé-bas:** 
$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

**passé-haut:** 
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$



## Filtres « rejet de bande »

**idéal:**

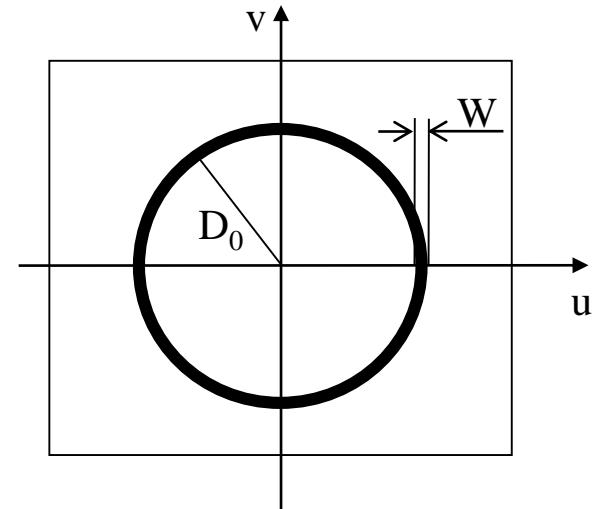
$$H(u,v) = \begin{cases} 1 & \text{si } D(u,v) < D_0 - \frac{W}{2} \\ 0 & \text{si } D_0 - \frac{W}{2} \leq D(u,v) \leq D_0 + \frac{W}{2} \\ 1 & \text{si } D_0 + \frac{W}{2} < D(u,v) \end{cases}$$

**Butterworth:**

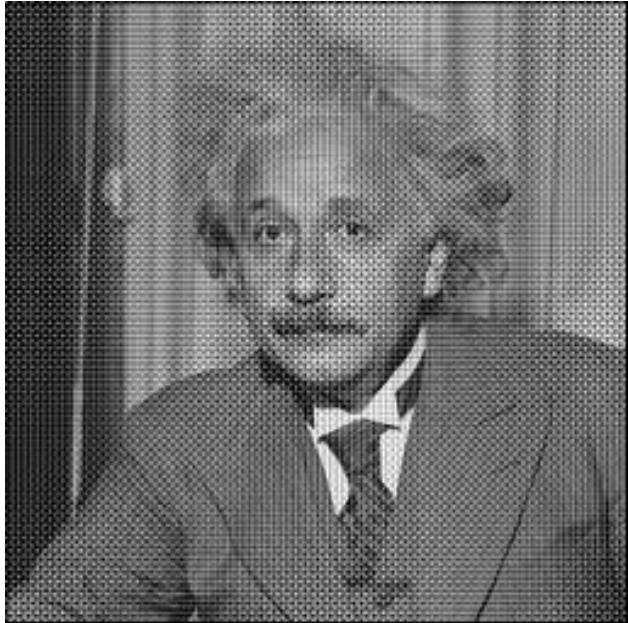
$$H(u,v) = \frac{1}{1 + \left[ \frac{D(u,v)W}{D^2(u,v) - D_0^2} \right]^{2n}}$$

**gaussien:**

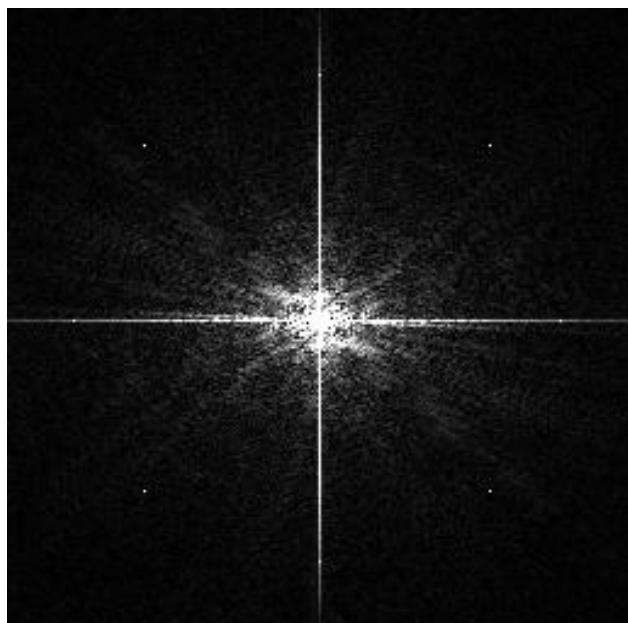
$$H(u,v) = 1 - e^{-\frac{1}{2} \left[ \frac{D^2(u,v) - D_0^2}{D(u,v)W} \right]}$$



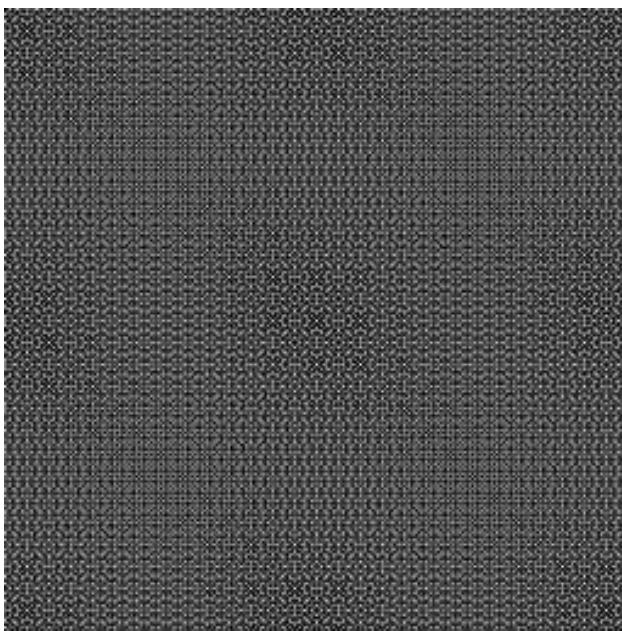
*Image bruitée*



*Spectre d'amplitude*



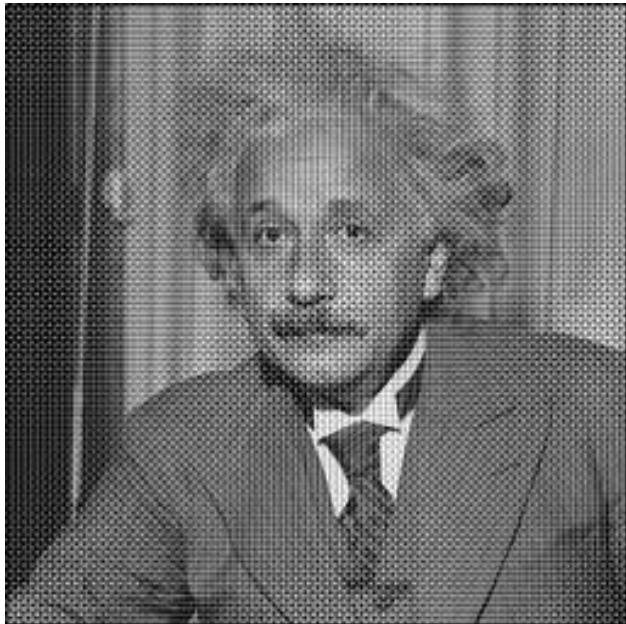
*Image du bruit*



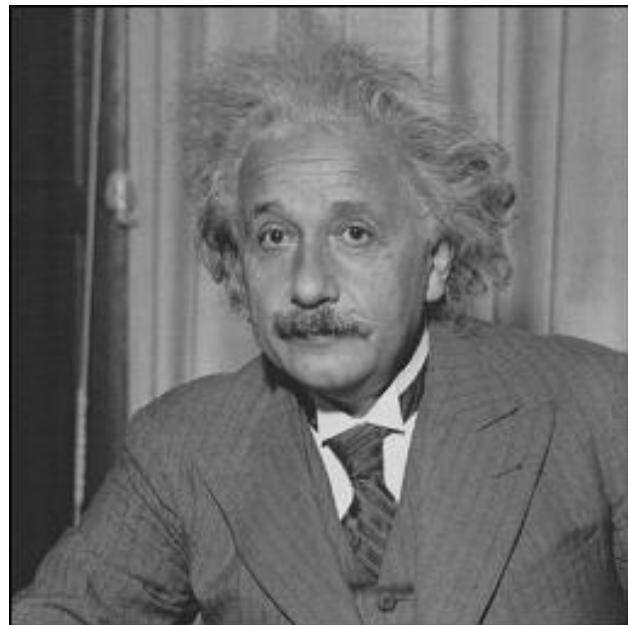
*Spectre du bruit*



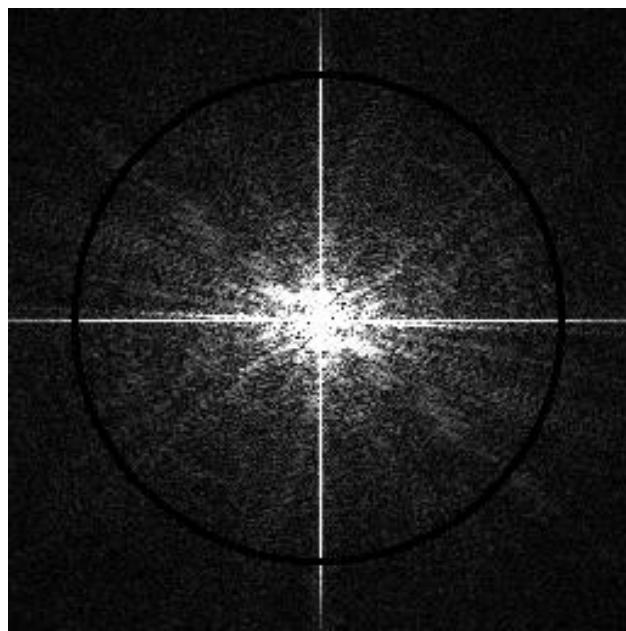
*Image bruitée*



*Image originale*



*Spectre filtré*



*Image filtrée*

